## MATH

MATH Co-Processor
XMC ${ }^{\text {TM }}$ microcontrollers
September 2016

## Agenda

Overview

Key feature: 32-bit divide
Key feature: Trigonometric functions
Key feature: Vector rotation (Park transform)
System integration
Result chaining between Divider \& CORDIC

Benchmarking results

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## MATH <br> MATH Co-Processor

## Highlights

The MATH Co-Processor provides a 32-bit signed or unsigned divider as well as a 24-bit CORDIC for trigonometric calculations. Both DIVIDER and CORDIC can operate in parallel next to the CORTEX ${ }^{\circledR}-\mathrm{MO}$ CPU core.

## Customer benefits

, The calculation time of a divide operation is reduced to $50 \%$
, Increase of computational power for real time critical tasks
, Field oriented motor control algorithms are implemented with high resolution

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## MATH 32-bit divide

, Signed/unsigned 32-bit division in 35 kernel clock cycles
, Operands pre-processing with configurable number of:

- Left shifts for dividend
- Right shifts for divisor
, Result post-processing with configurable number of shifts and shift direction


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## MATH

## Trigonometric functions (1/2)

| Function | Rotation Mode | Vectoring Mode |
| :---: | :---: | :---: |
|  | $d_{\mathrm{i}}=\operatorname{sign}\left(z_{\mathrm{i}}\right), z_{\mathrm{i}} \rightarrow 0$ | $d_{\mathrm{i}}=-\operatorname{sign}\left(y_{\mathrm{i}}\right), y_{\mathrm{i}} \rightarrow 0$ |
| Circular $\begin{aligned} & m=1 \\ & e_{\mathrm{i}}=\operatorname{atan}\left(2^{-1}\right) \end{aligned}$ | $\begin{aligned} & X_{\text {final }}=\mathrm{K}[X \cos (Z)-Y \sin (Z)] / \mathrm{MPS} \\ & Y_{\text {final }}=\mathrm{K}[Y \cos (Z)+X \sin (Z)] / \mathrm{MPS} \\ & Z_{\text {final }}=0 \\ & \text { where } \mathrm{K} \approx 1.646760258121 \end{aligned}$ | $\begin{aligned} & X_{\text {final }}=\mathrm{K} \operatorname{sqrt}\left(X^{2}+Y^{2}\right) / \mathrm{MPS} \\ & Y_{\text {final }}=0 \\ & Z_{\text {final }}=Z+\operatorname{atan}(Y / X) \\ & \text { where } \mathrm{K} \approx 1.646760258121 \end{aligned}$ |
| $\begin{aligned} & \text { Linear } \\ & m=0 \\ & e_{\mathrm{i}}=2^{-1} \end{aligned}$ | $\begin{aligned} & X_{\text {final }}=X / \mathrm{MPS} \\ & Y_{\text {final }}=[Y+X Z] / \mathrm{MPS} \\ & Z_{\text {final }}=0 \\ & \hline \end{aligned}$ | $\begin{aligned} & X_{\text {final }}=X / \mathrm{MPS} \\ & Y_{\text {final }}=0 \\ & Z_{\text {final }}=Z+Y / X \\ & \hline \end{aligned}$ |
| $\begin{aligned} & \text { Hyperbolic } \\ & m=-1 \\ & e_{i}=\operatorname{atanh}\left(2^{-1}\right) \end{aligned}$ | $\begin{aligned} & X_{\text {final }}=\mathrm{k}[X \cosh (Z)+Y \sinh (Z)] / \\ & \text { MPS } \\ & Y_{\text {final }}=\mathrm{k}[Y \cosh (Z)+X \sinh (Z)] / \\ & \text { MPS } \\ & Z_{\text {final }}=0 \\ & \text { where } \mathrm{k} \approx 0.828159360960 \end{aligned}$ | $\begin{aligned} & \hline X_{\text {final }}=\mathrm{k} \operatorname{sqrt}\left(X^{2}-Y^{2}\right) / \mathrm{MPS} \\ & Y_{\text {final }}=0 \\ & Z_{\text {final }}=Z+\operatorname{atanh}(Y / X) \\ & \text { where } \mathrm{k} \approx 0.828159360960 \end{aligned}$ |

To calculate sin(angle) and $\cos$ (angle)
, Setup function to "Circular", "Rotation Mode"
, $\mathrm{X}=1 / \mathrm{K}, \mathrm{Y}=0, \mathrm{Z}=$ "angle"
, Result_X $=\cos ($ angle $)$
, Result_Y = sin(angle)

## MATH

## Trigonometric functions (2/2)

| Function | Rotation Mode | Vectoring Mode |
| :---: | :---: | :---: |
|  | $d_{\mathrm{i}}=\operatorname{sign}\left(z_{\mathrm{i}}\right), z_{\mathrm{i}} \rightarrow 0$ | $d_{\mathrm{i}}=-\operatorname{sign}\left(y_{i}\right), y_{i} \rightarrow 0$ |
| Circular $\begin{aligned} & m=1 \\ & e_{\mathrm{i}}=\operatorname{atan}\left(2^{-1}\right) \end{aligned}$ | $\begin{aligned} & X_{\text {final }}=\mathrm{K}[X \cos (Z)-Y \sin (Z)] / \mathrm{MPS} \\ & Y_{\text {final }}=\mathrm{K}[Y \cos (Z)+X \sin (Z)] / \mathrm{MPS} \\ & Z_{\text {final }}=0 \\ & \text { where } \mathrm{K} \approx 1.646760258121 \end{aligned}$ | $\begin{aligned} & X_{\text {final }}=\mathrm{K} \operatorname{sqrt}\left(X^{2}+Y^{2}\right) / \mathrm{MPS} \\ & Y_{\text {final }}=0 \\ & Z_{\text {final }}=Z+\operatorname{atan}(Y / X) \\ & \text { where } \mathrm{K} \approx 1.646760258121 \end{aligned}$ |
| $\begin{aligned} & \text { Linear } \\ & m=0 \\ & e_{\mathrm{i}}=2^{-1} \end{aligned}$ | $\begin{aligned} & X_{\text {final }}=X / \mathrm{MPS} \\ & Y_{\text {final }}=[Y+X Z] / \mathrm{MPS} \\ & Z_{\text {final }}=0 \end{aligned}$ | $\begin{aligned} & X_{\text {final }}=X / \mathrm{MPS} \\ & Y_{\text {final }}=0 \\ & Z_{\text {final }}=Z+Y / X \\ & \hline \end{aligned}$ |
| $\begin{aligned} & \hline \text { Hyperbolic } \\ & m=-1 \\ & e_{\mathrm{i}}=\operatorname{atanh}\left(2^{-1}\right) \end{aligned}$ | $\begin{aligned} & \hline X_{\text {final }}=\mathrm{k}[X \cosh (Z)+Y \sinh (Z)] / \\ & \text { MPS } \\ & Y_{\text {final }}=\mathrm{k}[Y \cosh (Z)+X \sinh (Z)] / \\ & \text { MPS } \\ & Z_{\text {final }}=0 \\ & \text { where } \mathrm{k} \approx 0.828159360960 \end{aligned}$ | $\begin{aligned} & \hline X_{\text {final }}=\mathrm{k} \operatorname{sqrt}\left(X^{2}-Y^{2}\right) / \mathrm{MPS} \\ & Y_{\text {final }}=0 \\ & Z_{\text {final }}=Z+\operatorname{atanh}(Y / X) \\ & \text { where } \mathrm{k} \approx 0.828159360960 \end{aligned}$ |

To calculate $\arctan (\mathrm{Y} / \mathrm{X})$
, Setup function to "Circular", "Vectoring Mode"
, $Z=0$
, Result_X $=\mathrm{K} \operatorname{sqrt}\left(\mathrm{X}^{2}+\mathrm{Y}^{2}\right)$
, Result_Z $=\arctan (\mathrm{Y} / \mathrm{X})$

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## MATH

## Vector rotation (Park transform)

| Function | Rotation Mode | Vectoring Mode |
| :---: | :---: | :---: |
|  | $d_{\mathrm{i}}=\operatorname{sign}\left(z_{\mathrm{i}}\right), z_{\mathrm{i}} \rightarrow 0$ | $d_{\mathrm{i}}=-\operatorname{sign}\left(y_{\mathrm{i}}\right), y_{\mathrm{i}} \rightarrow 0$ |
| Circular $\begin{aligned} & m=1 \\ & e_{\mathrm{i}}=\operatorname{atan}\left(2^{-1}\right) \end{aligned}$ | $\begin{aligned} & X_{\text {final }}=\mathrm{K}[X \cos (Z)-Y \sin (Z)] / \mathrm{MPS} \\ & Y_{\text {final }}=\mathrm{K}[Y \cos (Z)+X \sin (Z)] / \mathrm{MPS} \\ & Z_{\text {final }}=0 \\ & \text { where } \mathrm{K} \approx 1.646760258121 \end{aligned}$ | $\begin{aligned} & X_{\text {final }}=\mathrm{K} \operatorname{sqrt}\left(X^{2}+Y^{2}\right) / \mathrm{MPS} \\ & Y_{\text {final }}=0 \\ & Z_{\text {final }}=Z+\operatorname{atan}(Y / X) \\ & \text { where } \mathrm{K} \approx 1.646760258121 \end{aligned}$ |
| Linear $\begin{aligned} & m=0 \\ & e_{\mathrm{i}}=2^{-\mathrm{i}} \end{aligned}$ | $\begin{aligned} & \hline X_{\text {final }}=X / \mathrm{MPS} \\ & Y_{\text {final }}=[Y+X Z] / \mathrm{MPS} \\ & Z_{\text {final }}=0 \\ & \hline \end{aligned}$ | $\begin{aligned} & X_{\text {final }}=X / \mathrm{MPS} \\ & Y_{\text {final }}=0 \\ & Z_{\text {final }}=Z+Y / X \\ & \hline \end{aligned}$ |
| Hyperbolic $\begin{aligned} & m=-1 \\ & e_{\mathrm{i}}=\operatorname{atanh}\left(2^{-1}\right) \end{aligned}$ | $\begin{aligned} & X_{\text {final }}=\mathrm{k}[X \cosh (Z)+Y \sinh (Z)] / \\ & \text { MPS } \\ & Y_{\text {final }}=\mathrm{k}[Y \cosh (Z)+X \sinh (Z)] / \\ & \text { MPS } \\ & Z_{\text {final }}=0 \\ & \text { where } \mathrm{k} \approx 0.828159360960 \end{aligned}$ | $\begin{aligned} & \hline X_{\text {final }}=\mathrm{k} \mathrm{sqrt}\left(X^{2}-Y^{2}\right) / \mathrm{MPS} \\ & Y_{\text {final }}=0 \\ & Z_{\text {final }}=Z+\operatorname{atanh}(Y / X) \\ & \text { where } \mathrm{k} \approx 0.828159360960 \end{aligned}$ |

Park transform

$$
\begin{aligned}
& \mathrm{i}_{\mathrm{d}}=\mathrm{i}_{\alpha} \cos \varphi+\mathrm{i}_{\beta} \sin \varphi \\
& \mathrm{i}_{\mathrm{q}}=-\mathrm{i}_{\alpha} \sin \varphi+\mathrm{i}_{\beta} \cos \varphi
\end{aligned}
$$

To calculate Park transform
, Setup function to "Circular", "Rotation Mode"
, $X=i_{\beta}, Y=i_{\alpha}, Z=\varphi$
, Result_X = iq
, Result_Y = id

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## MATH

## System integration


, Target applications

- Motor control
- Intelligent lighting
- Power conversion


The math co-processor can be clocked with a frequency of up to 64 MHz and is accessible via the SFR interface. The sub-blocks, a 32-bit divider and a 24-bit CORDIC can be used next to the CPU independently. The execution of the math unit can be configured to be twice the MCU clock. Hence a divide is executed in 18 CPU clocks and a CORDIC function takes up to 31 CPU clocks.

In some use cases, the result of one sub-block is needed as data input for the other subblock. A hardware mechanism is provided for autonomous execution of both calculation with result chaining.

The result that is read from the SFR-interface is always provided as the latest result after processing the math unit's command. In case the read instruction is executed while the math is still busy, the bus-interface will add wait states until the latest result is available.

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## MATH <br> Result chaining between Divider \& CORDIC (1/6)

CORDIC's result to DIV's input
, Result of the CORDIC operation can be "forward" directly to the Divider operand register, DVD and DVS

DIV's result to CORDIC's input
, QUOT and RMD result can be "forward" directly to the CORDIC operand register, CORD[Z:X]

## MATH

## Result chaining between Divider \& CORDIC (2/6)

## Global Control Register (GLBCON)



[^0]CORDXRC
CORDYRC

## MATH <br> Result chaining between Divider \& CORDIC (3/6)

The next few slides illustrate a simple example for result chaining


After the computation of the CORDIC operation, the result will be written to CORRX. This result will also be written to DIV's DVS.

## MATH <br> Result chaining between Divider \& CORDIC (4/6)

DVS (Divisor Register)


rwh
, As the 24-bit CORDIC result is assigned to bit[8 to 31], it might be necessary for some pre-processing of the input value before the DIV operation
, DIVCON.DVSSRC - right shift the input value before the division operation

Value at DVS


DVSSRC = 8
Right shift by 8
Value use for the Division operation


Result chaining between Divider \& CORDIC (5/6)

| Function | Vectoring Mode |
| :--- | :--- |
|  | $d_{\mathrm{i}}=-\operatorname{sign}\left(y_{\mathrm{i}}\right), y_{i} \rightarrow 0$ |
| Circular | $X_{\text {final }}=\mathrm{K} \operatorname{sqrt}\left(X^{2}+Y^{2}\right) / \mathrm{MPS}$ |
| $m=1$ | $Y_{\text {final }}=0$ |
| $e_{\mathrm{i}}=\operatorname{atan}\left(2^{-1}\right)$ | $Z_{\text {final }}=Z+\operatorname{atan}(Y / X)$ |
|  | where $\mathrm{K} \approx 1.646760258121$ |

, CORDIC is setup to Circular Vectoring Mode
, CORDIC will start when CORDX is written
, DIV will start when DVS is written
, The result of CORDIC's CORRX will also update DIV's DVS with the same value
, This action will trigger the DIV operation to start
, The DIV's post-processing compensated for the difference in bit length of CORDIC(24-bit) and DIV(32-bit)
, As a result, the writing of CORDIC's CORDX orderly start both CORDIC and DIV

## Result chaining between Divider \& CORDIC (6/6)

```
GLBCON = 0x0003;
    // DVSRC = 011b;
DIVCON = 0x08000000;
    // ST_MODE = 0;
    // DVSSRC = 8;
DVD = 0x12345678;
CON = 0x0020;
    // MODE = 01b;
    // ROTVEC = 0;
    // ST_MODE = 0;
CORDY = (0x5678<<8);
CORDX = (0x1234<<8);
```

// DVS result will be updated when
// CORRX has new result
// Auto-Start when DVS is written
// DVS value will be shifted right by 8
// Preload the Dividend value first
// Circular Mode
// Vectoring Mode
// Auto-Start when CORDX is written
// Load Y parameter
// Load X parameter and start CORDIC
// Result Chain to DIV's DVS will auto start DIV

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## Benchmarking results

## MATH

, Execution time of a division operation and a cosine operation running on the MATH library is benchmarked against that of a similar operation running on standard C library
, Conditions:

- Execution time refers to complete function execution, inclusive of coprocessor configuration, writing of operands and state checking
- Ratio of PCLK to MCLK is 2:1
- Compliers from IFX, Keil and IAR were used


## MATH Benchmarking results (2/2)

, The benchmarking results are shown in the table below:

| Compiler |  | Division <br> (MCLK cycles) |  | Cosine <br> (MCLK cycles) |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | With <br> MATH LIB | With <br> Std C LIB | With <br> MATH LIB | With <br> Std C LIB |  |
| IAR EWARM v7.10 | 99 | 712 | 234 | 4574 |  |
| Keil $\mu$ Vision v5.10 | 95 | 230 | 238 | 6514 |  |
| DAVE $^{\text {TM }}$ v3.1.10 | 114 | 415 | 258 | 9832 |  |

, Significant performance boosts are seen when using MATH library over standard C library:

- ~ 7x performance for division
- ~ 38x performance for cosine


## General information

, For latest updates, please refer to: www.infineon.com/xmc1000
, For support: http://www.infineonforums.com/forums/8-XMC-Forum

## Support material



## Disclaimer

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[^0]:    Dividend Register Result Chaining
    The DVD register in DIV will be updated with the selected result register value when the result
    chaining trigger event occurs.
    $000_{\mathrm{B}}$ No result chaining is selected
    $001_{\mathrm{B}}$ QUOT register is the selected source
    $010_{B}$ RMD register is the selected source
    $011_{\mathrm{B}}$ CORRX is the selected source
    $100_{\mathrm{B}}$ CORRY is the selected source
    $101_{\mathrm{B}}$ CORRZ is the selected source
    CORDX Register Result Chaining
    The CORDX register in CORDIC will be updated with
    the selected result register value when the result
    chaining trigger event occurs.
    $00_{B}$ No result chaining is selected
    $01_{\mathrm{B}}$ QUOT register is the selected source
    $10_{B}$ RMD register is the selected source

